Semester 2 (Units 3 and 4) Examination, 2016

Question/Answer Booklet

MATHEMATICS METHODS

Section One: Calculator-free

Student Name/Number: _____

Teacher Name: _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor: This Question/Answer Booklet
Formula Sheet

To be provided by the candidate:

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	10	10	50	54	35
Section Two: Calculator-assumed	13	13	100	104	65
					100

Instructions to candidates

1. The rules for the conduct of School exams are detailed in the _____*School/College assessment policy*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

Section One: Calculator-free **(54 Marks) Weighting 35%**

This section has **10 (ten)** questions. Answer **all** questions. Write your answers in the spaces provided.

Suggested working time: **50 minutes**.

Question 1 **(6 marks)**

Determine the exact value of m, $m > 0$, for each of the following equations.

(a) $2\ln m = 3$ (2 marks)

(b) $\log(m+3) + \log m - 1 = 0$ (4 marks)

See next page

Question 2 **(9 marks)**

(a) Differentiate each of the following with respect to x. Do **not** simplify your answers.

(i) $y = \dfrac{4e^x}{6x^4 - x^3 + e}$ (3 marks)

(ii) $y = \ln\left(\dfrac{5x^3 + 3}{\sin(x)}\right)$ (3 marks)

(b) Show how to use the chain rule to determine $\dfrac{dy}{dx}$ when $y = \dfrac{e^{x^2 - \cos(x)}}{2}$ (3 marks)

See next page

Question 3 **(3 marks)**

Describe each of the following as either a discrete random variable, a continuous random variable or a non-random variable.

(a) the number of dots showing on a die after being thrown. (1 mark)

(b) the distance between Sydney and Melbourne. (1 mark)

(c) the thickness of wire coming off a production line. (1 mark)

Question 4 **(4 marks)**

Determine the value of k if $f(x)$ represents a probability density function.

$$f(x) = \begin{cases} kx\left(1 - \frac{x^2}{3}\right), & 0 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

See next page

Question 5 **(4 marks)**

The probability density function for a Bernoulli distribution is:

$$P(X = x) = \begin{cases} 1 - p \text{ , for } x = 0 \\ p \quad \text{ , for } x = 1 \end{cases}$$

Given that the standard deviation for a particular Bernoulli distribution is $\frac{\sqrt{3}}{4}$, determine the value(s) of p.

Question 6 **(4 marks)**

Consider the graph of $g(x) = \ln(2x + 6) - 4$

(a) For what values of x is the function valid? (1 mark)

(b) Determine the x-coordinate of the point on $g(x)$ where the slope of the tangent is 4. (3 marks)

See next page

Question 7 **(4 marks)**

The probability density function of a discrete random variable Y is given by

$P(Y = y) = ky^2$, for $y = 0, 1, 2, 3, 4$.

(a) Complete the probability distribution for Y (2 marks)

y	0	1	2	3	4
$P(Y = y)$			$4k$		

(b) Determine the value of k. (2 marks)

Question 8 **(3 marks)**

Given $\int e^{f(x)} f'(x)\,dx = e^{f(x)}$. If $f'(x) = 2xe^{3x^2-1}$ and $f(0) = 0$ determine $f(x)$.

See next page

Question 9 **(12 marks)**

When calculating a confidence interval for a population proportion from a sample an associated z score is used. Use the table below to answer the following questions:

Confidence Interval	z score (rounded to 1 decimal place)
95%	2.0
87%	1.5
68%	1.0

(a) In a random sample of 100 people, 20 said they had watched an AFL game in the last year.

(i) Determine the proportion of those in the sample who had watched an AFL game in the last year (1 mark)

(ii) Determine a 95% confidence interval for the proportion of the population who had watched an AFL game in the last year. (4 marks)

A random sample of size n_1 was taken and the proportion of people who had watched a game of AFL in the last year was m.

(b) Determine a 68% confidence interval for the proportion of the population who had watched an AFL game in the last year in terms of n_1 and m. (2 marks)

See next page

(c) A new sample of size n_2 was taken and the proportion of people who had watched a game of AFL in the last year was again m. When an 87% confidence interval was determined it was found to be the same as the interval determined in part (b).

(i) Is n_2 larger or smaller than n_1? Explain (2 marks)

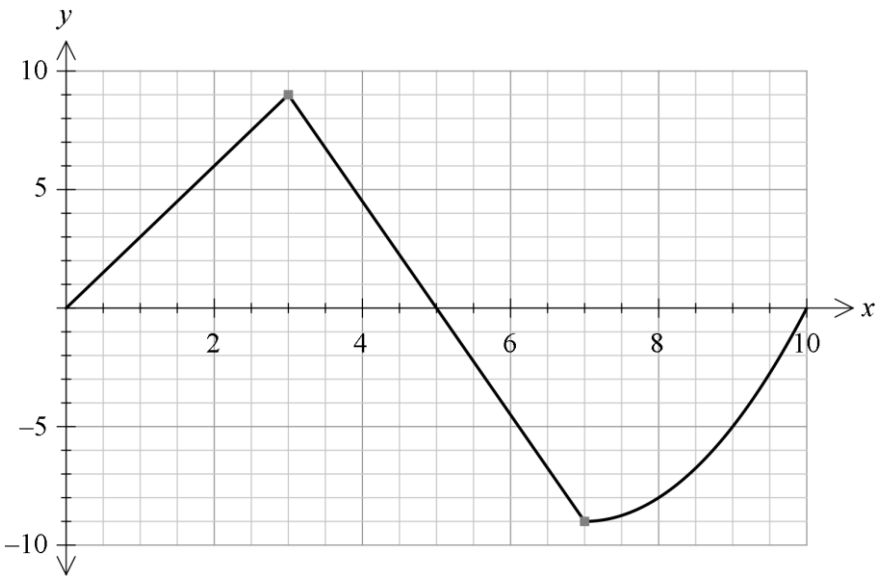
(ii) What is the relationship between n_1 and n_2?

(3 marks)

See next page

Question 10 **(5 marks)**

The graph of $y = f(x)$ is shown below. It consists of two straight lines followed by a curve. The area between the function and the x-axis is equal to 50 square units.



(a) $\int_0^5 f(x)dx$ (2 marks)

(b) $\int_7^{10} f(x)dx$ (3 marks)

End of Questions

 See next page

Additional working space

Question number: _____

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